Some references for the eigen-semester lecture series

February 15, 2006

In anticipation of the lectures to be given this Spring semester, we have begun to collect a list of titles books and articles some to serve as preparatory reading, some as background reading, and some as references for the lectures themselves. We will be up-dating this schedule as the semester approaches. This information is also on the web:

`http://www.math.harvard.edu/ev/`

along with the full calendar of events.

Kevin Buzzard will be giving a topics course (Math 255y) "A concrete approach to p-adic modular forms." Tu, Th 1:00-2:230. Barry Mazur's topics course (Math 254z) will be partly devoted to preparatory material for some of the lecture series scheduled, and partly used as a time-slot for some of those lecture series.

The lecture series by Jacques Tilouine on p-adic Siegel modular forms.

First Lecture: Monday, Feb 6 2-3 p.m.

Bibliography:

- Haruzo Hida's paper "p-Adic automorphic forms on reductive groups," in Astérisque **298** (2005),
- Chapter 3 of Hida's book "p-Adic automorphic forms on Shimura Varieties" Springer Monographs in Mathematics (thismakes free use of Chapt.4of Faltings-Chai's book: Degeneration of Abelian Varieties, Erg. Math. Wiss, Springer Verlag)
- Hida's paper in J. Inst. Math. Jussieu **1** (2002)
- Johan de Jong, "The moduli spaces of principally polarized abelian varieties with $\Gamma_0(p)$-level structure," Journal of Algebraic Geometry, **2** (1993), pp. 667-688.

- Also, a series of papers and preprints of Tilouine that will be relevant during the lectures, and available then.

The lecture series by Peter Schneider on p-adic Banach representations

First Lecture: Wednesday, March 8 2-3 p.m.

Outline and Bibliography: Professor Schneider writes:

Lecture 1: Motivation

I will discuss basic structural features of the absolute Galois group of a local number field, will recall the shape of the classical local Langlands correspondence, and will review the beginnings of the theory of p-adic Galois representations.

Lecture 2: Banach space representations

For any p-adic Lie group I will construct an abelian category of p-adic Banach space representations. As a basic example I will discuss the continuous principal series for reductive p-adic groups.

Lecture 3: p-adic Satake isomorphism

I will introduce certain Banach algebra completions of the Hecke algebra of a maximal compact subgroup in a split p-adic reductive group and will compute them explicitly as algebras of analytic functions on an affinoid in the dual torus.

Lecture 4: Unramified p-adic functoriality

Here I will describe the emerging picture of a correspondence between crystalline p-adic Galois representations and characters of the p-adic Banach algebras of the last lecture.

Optional preparatory reading:

1. Borel A.: Automorphic L-functions. In Proc. Symp. Pre Math. **33**(2), pp. 27-61. AMS 1979 (only Chapters I and II !)
2. Deligne P., Milne J.S.: Tannakian categories. Springer LNM **900**, pp. 101- 228 (only neutral Tannakian categories)
3. Fontaine J.-M.: Representations ℓ-adiques potentiellement semi-stables. Astérisque **223**, 321-347 (1994)
4. Schneider P.: Nonarchimedean functional analysis. Springer 2001
5. Tate J.: Number theoretic background. In Proc. Symp. Pre Math. **33**(2), pp. 3-26. AMS 1979

The lecture series by Gaetan Chenevier "The eigenvarieties of definite unitary groups" (with a focus on non-ordinary aspects)

Gaetan Chenevier has written this description of his lecture series:

Lecture 1, 2: Trianguline deformation problems for crystalline representations (jont work with J.Bellaiche).

Abstract: I will define and study some deformation problems of the n-dimensional crystalline representations of the absolute Galois group of $\mathbf{Q}_p$. I will explain some results on trianguline representations over artinian rings (and of any rank) extending some previous work of Colmez. As an application, I will explain how to "predict" Galois theoretically some aspects of the theory of p-adic families of automorphic forms.

References:

- Mark Kisin's "Overconvergent modular forms and the Fontaine-Mazur conjecture", Inv. Math **153(2)**
- Laurent Berger's "Representations p-adiques et équations différentielles", Inv. Math **148**.
- Pierre Colmez's "Série principale unitaire pour $\mathrm{GL}_2(\mathbf{Q}_p)$ et représentations triangulines de dimension 2", available on his web page.
- My preprint with J. Bellaiche on my web page (to appear very soon if not already there).

Lecture 3, 4, 5: The p-adic family of algebraic representations of $\mathrm{GL}_n(\mathbf{Q}_p)$. Definite unitary groups over number fields, their automorphic forms and eigenvarieties.

Abstract: I will explain the example of the p-adic families of the algebraic representations of $\mathrm{GL}_n(\mathbf{Q}_p)$. After introducing unitary groups over $\mathbf{Q}$ and discussing their automorphic forms, I will show how this example leads to a simple construction of the eigenvarieties of the definite unitary groups.

References :

- For automorphic forms, - A.Borel, "Automorphic L-functions", PSPM **33 (2)**.
- Rogawski's articles in Langlands-Ramakhrishnan's book "The zeta function of Picard modular surfaces", Publ. CRM Montreal.
- Harris-Taylors's book "The cohomology of some simple Shimura varieties", Annals of math. studies **151**. For p-adic modular forms,
- Robert Coleman's "p-adic Banach spaces and families of modular forms", Inv. math. **127**.
- Kevin Buzzard's "Eigenvarieties", available on his web page.

- My paper ”Familles p-adiques de formes automorphes pour $\mathbf{GL}_n$”, Crelle's Journal **570**.

Lecture 6, 7: The Galois pseudorepresentation on eigenvarieties. Local geometry of unitary eigenvarieties at some special points and Selmer groups (joint work with J.Bellaiche).

The lecture series by Mark Kisin "The eigencurve via Galois representations"

First Lecture: Monday, March 20 2-3 p.m.

Program of lectures:

1. Families of finite slope representations.

   Reading:

   - Coleman-Mazur: The Eigencurve, in *Galois representations in Arithmetic Algebraic Geometry (Durham 1996)* London Math Soc Lecture Notes **154** (1997) 1-113
   - Mark Kisin: ”Overconvergent modular forms and the Fontaine-Mazur conjecture. Invent. Math. **153**(2) (2003), 373-454

2. Modularity and patching local rings over global ones.

   Reading: Sections 3.2-3.5 of the preprint "Moduli of finite flat group schemes and modularity" on Kisin's homepage.

3. Existence of semi-stable deformation rings and the Breuil-Mezard conjecture.

   Reading: For the first part of this topic there should soon be a preprint. The construction uses the theory of the paper ”Crystalline representations and F-crystals" which is on Kisin's webpage.

4. The Fontaine-Mazur conjecture via the p-adic local Langlands correspondence.

The series of lectures by Chris Skinner.

First meeting: Wednesday March 29 2-3 p.m.

Chris Skinner writes: "Probably the most important thing is to understand the structure of Ribet's paper and how Wiles incorporated it with Hida familes to prove the Main Conjecture. The rest is just for those interested in details of the techniques." Specific references for the specific topics to be covered are:

1. *General Iwasawa theory:*

   (a) Greenberg's papers, esp.
   "Iwasawa theory for p-adic representations." Algebraic number theory, 97–137, Adv. Stud. Pure Math., 17, Academic Press, Boston, MA, 1989. (Reviewer: Karl Rubin) 11R23 (11G40 11R34)

(b) "Iwasawa theory for motives. L-functions and arithmetic." (Durham, 1989), 211–233, London Math. Soc. Lecture Note Ser., **153**, Cambridge Univ. Press, Cambridge, 1991.

(c) "Iwasawa theory and p-adic deformations of motives." Motives (Seattle, WA, 1991), 193–223, Proc. Sympos. Pure Math., **55**, Part 2, Amer. Math. Soc., Providence, RI, 1994.

2. *for the* GL(1)/GL(2) *cases:*

   (a) Ribet, Kenneth A. "A modular construction of unramified p-extensions of $\mathbf{Q}(\mu_p)$." Invent. Math. **34** (1976), no. 3, 151–162.

   (b) Wiles, A. "The Iwasawa conjecture for totally real fields." Ann. of Math. (2) **131** (1990), no. 3, 493–540.

   (c) Wiles, A. "On p-adic representations for totally real fields." Ann. of Math. (2) **123** (1986), no. 3, 407–456.

3. *for Hida theory:*

   Hida's papers, esp.

   "p-adic automorphic forms on reductive groups." Automorphic forms. I. Astrisque No. **298** (2005), 147–254.(also available on his webpage)

4. *for Galois representations for unitary groups:*

   - See the example of unitary groups in
     Blasius, Don; Rogawski, Jonathan D. Zeta functions of Shimura varieties. Motives (Seattle, WA, 1991), 525–571, Proc. Sympos. Pure Math., **55**, Part 2, Amer. Math. Soc., Providence, RI, 1994.
   - Also see the relevant section(s) of
     Harris, Michael: "L-functions and periods of polarized regular motives." J. Reine Angew. Math. **483** (1997), 75–161.

5. *or the Eisenstein series on unitary groups see (at your own risk):*

   Shimura, Goro Euler products and Eisenstein series. CBMS Regional Conference Series in Mathematics, **93**.

   esp. chapter 12 and chapters 20-22.

The series of lectures by Matthew Emerton's on "Local-global compatibility in the p-adic Langlands correspondence for GL_2 over Q"

First meeting: Monday, April 10 1-2 p.m.

The lecture series by Laurent Berger on some aspect of the p-adic Langlands program:

First Lecture: Monday, April 10 2-3 p.m.

The lecture series by Christophe Breuil: "Aspects of p-adic Langlands correspondence I, II, III and IV"

First Lecture: Tuesday, April 11 2:30-4:00 p.m.

Part of the first lecture would be the history and precise statement of the conjecture and then will go onto an examination of some p-adic Langlands considerations for local representations that are potentially semi-stable."

For books:

- Astérisque **223** *Périodes p-adiques,* especially (pp. 321-348) the article "Repréesentations ℓ-adiques potentiellement semi-stables" by Jean-Marc Fontaine,
- Steven Gelbart's book "Automorphic forms on adele groups" (Princeton Univ. Press)
- Daniel Bump's book "Automorphic forms and representations" (Cambridge studies in Pure Maths **55**).

For articles:

- Laurent Berger's survey on p-adic Hodge theory (can be found on his home page)
- "Modular forms and p-adic Hodge theory" by T. Saito (Inv. Math. **129**, 607-620)
- Peter Schneider and Jeremy Teitelbaum's papers (there are 3 or 4 the references can be found on P. Schneider's home page in Muenster)
- Steven Kudla's survey on Local Langlands (Motives, Seattle, AMS)
- Matthew Emerton's survey "A local-global compatibility in the p-adic Langlands programme for $GL_2/\mathbf{Q}$" (available from his home page)

The lecture series by Chandrasekhar Khare on Serre's Conjecture.

First Lecture: Friday, April 14 1-2 p.m.

The lecture series by Michael Harris: "On the stable trace formula for unitary groups with applications to construction of Galois representations:"

First Lecture: Friday, April 14 2-3 p.m.

Bibliography:

- See the texts at the following website:

  `http://www.institut.math.jussieu.fr/projets/fa/bpRef.html`

  especially items 1 and 5.

- The article of Blasius and Rogawski: "Motives for Hilbert modular forms," Inventiones **114** (1993) contains nearly complete results for $GL(2)$ and could serve as an introduction to the general problem.

The series of lectures by Haruzo Hida, "L-invariant and Galois deformation theory"

First Lecture: Wednesday, April 26 2-3 p.m.

Professor Hida writes:

I will try to describe the following topics on Hilbert modular forms:

1. Theory of p-adic ordinary families of Hilbert modular forms;
2. Identification of Galois deformation rings and Hecke algebras;
3. Computation of Greenberg's L-invariant of standard and adjoint square L-functions.

I have posted in my graduate course web page "Math 207C Winter 2006" (www.math.ucla.edu/ hida) a part of the manuscript of the first two chapters of my forthcoming book "Hilbert Modular Forms and Iwasawa Theory" from Oxford University Press, which will serve as an introduction to my lectures. Clicking the link "Math 207c Winter 2006" in my homepage (as indicated above), one can get into the class homepage of the graduate course in which one can find links to the book manuscript. The course notes will be also posted in the above class homepage.

As for other references, my two earlier books will be good:

1. Chapter 7 of "Elementary Theory of L-Functions and Eisenstein Series," LMSST **26**, Cambridge University Press, 1993.
2. Chapters 3 and 5 of "Modular Forms and Galois Cohomology," Cambridge Studies in Advanced Mathematics **69**, Cambridge University Press, 2000