Spring 2016 Tutorial
Morse Theory

Description Morse theory is the study of the topology of smooth manifolds by looking at smooth functions. It turns out that a “generic” function can reflect quite a lot of information of the background manifold.

In Morse theory, such “generic” functions are called “Morse functions”. By definition, a Morse function on a smooth manifold is a smooth function whose Hessians are non-degenerate at critical points. One can prove that every smooth function can be perturbed to a Morse function, hence we think of Morse functions as being “generic”.

Roughly speaking, there are two different ways to study the topology of manifolds using a Morse function. The classical approach is to construct a cellular decomposition of the manifold by the Morse function. Each critical point of the Morse function corresponds to a cell, with dimension equals the number of negative eigenvalues of the Hessian matrix. Such an approach is very successful and yields lots of interesting results. However, for some technical reasons, this method cannot be generalized to infinite dimensions. Later on people developed another method that can be generalized to infinite dimensions. This new theory is now called “Floer theory”.

In the tutorial, we will start from the very basics of differential topology and introduce both the classical and Floer-theory approaches of Morse theory. Then we will talk about some of the most important and interesting applications in history of Morse theory. Possible topics include but are not limited to: Smooth h-Cobordism Theorem, Generalized Poincare Conjecture in higher dimensions, Lefschetz Hyperplane Theorem, and the existence of closed geodesics on compact Riemannian manifolds, and so on. If students are interested, we can also talk about other closely related topics such as the Conley index theory, Morse-Bott theory, equivariant homology, or Bott periodicity.

The background and interest of sstudents will be the first priority when choosing topics for the tutorial.

Suggested Pre-requisitions: *Although we will briefly review the basic terminologies of manifolds, it is highly recommended that the students have some familiarity with the concepts of smooth manifolds, tangent and cotangent bundles, and tangent maps. Prior knowledge in homology theory is not required but would be helpful*

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Partitions, Young Diagrams and Beyond

Quotation: The theory of partitions is one of the very few branches of mathematics that can be appreciated by anyone who is endowed with little more than a lively interest in the subject. Its applications are found wherever discrete objects are to be counted or classified, whether in the molecular and the atomic studies of matter, in the theory of numbers, or in combinatorial problems from all sources. ***Gian-Carlo Rota***

Description: A partition of n is a finite weakly decreasing sequence of positive integers with a sum equal to n. It can be visualized as a Young diagram: a collection of cells arranged in left-justified rows with row lengths given by elements of the sequence.

Despite such elementary description, Young diagrams occur in a variety of interplays between combinatorial and algebraic structures, related in particular to group representation theory, algebra of symmetric polynomials, and beautiful identities with series. They lead to surprising connections; in particular the knowledge about Young diagrams and representations of the symmetric group sheds light on questions such as:

How many times do you need to shuffle a deck of cards to make it close to random?
or
What can we say about the length of a longest increasing subsequence of a random permutation?

Such interplays and their consequences will be the focus of this tutorial.

Prerequisites: *Only basic linear algebra and interest in combinatorics. All other notions will be introduced during the course.*

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